I. Cloze Tests

1. $3^i =$
2. $z_n = \left(\frac{1+i}{3}\right)^n + i\left(1+\frac{3}{n}\right)^n$
3. If C denotes the circle centered at z_0 positively oriented and n is a positive integer，then $\int_C \frac{1}{(z-z_0)^n} dz =$ ________ . $\int_C \frac{\sin z}{(z-z_0)^n} dz =$ ________
4. The radius of convergence of $\sum_{n=1}^{\infty}(3n^3+2n+1)z^n$ is ____________.
5. $\sum_{n=1}^{\infty}(n^2+1)z^n$, $\sum_{n=1}^{\infty}(3n^2-6)z^n$
6. The singular points of the function $f(z)=\frac{\cos^2 z}{z(z^2+3)}$ are ___________.
7. $f(z)=\frac{\sin z}{z(z^2+1)}$, $f(z)=\frac{\cos z + z^4}{z^4(z^2-2)}$
8. $\mathrm{Res}\left(\frac{\exp(z)}{z^{2n}},\ 0\right) =$ ______ , where n is a positive integer.
9. $\frac{d}{dz}(e^z \sin^3 z) =$ ____________________ . $\frac{d}{dz} e^z \cos^2 z = ?$
10. The main argument and the modulus of the number $1-i$ are __________.
11. The square roots of $1-i$ are ____________.
12. The definition of e^z is _______________. $\cos z$?
13. $\mathrm{Log}(1-i)$ =_________________________.
14. $\mathrm{Log}(1+i)$ =__________________
15. The solutions of the equation $e^{3zi}-1=0$ are ___________

II. Computations

（1）$\oint_{|z-a|=a} \frac{dz}{z^2-a^2}$ ，（2）$\oint_{|z|=1} \frac{dz}{z^2+2z+4}$ ，（3）$1^{\sqrt{2}}$ ，（4）$\oint_{|z|=2} \frac{zdz}{z-3}$ ，

（5）$\oint_{|z|=2} \frac{ze^z dz}{z^2-1}$ ，（6）$\oint_{|z|=3} \frac{dz}{(z-2)^2}$ 。

1. Find $\int_{|z|=1}\frac{5zdz}{(2z+1)(z-2)}$. $\int_{|z|=1}\frac{zdz}{(2z+1)(z-2)}$, $\int_{|z|=1}\frac{9zdz}{(3z+1)(3z+2)}$

2. Find the value of

$\int\limits_{|z|=1}\frac{e^z\sin^2 z}{z^2}dz+\int\limits_{|z|=2}\frac{z^8dz}{(1-z)^2}$. $\int\limits_{|z|=1}\frac{e^z\sin^2 z}{z^2}dz+\int\limits_{|z|=2}\frac{z^3dz}{(1-z)^2}$,

$\int\limits_{|z|=1}\frac{z^2-\sin z}{z^4}dz+\int\limits_{|z|=2}\frac{z^2dz}{(z^2-1)}$

3. Let $f(z)=\frac{z}{(z-1)(z-2)}$, $f(z)=\frac{3z^2}{(z+1)(z+2)}$ find the Laurent expansion of f on the annulus $D=\{z:0<|z|<1\}$.

4. Given $f(z)=\int\limits_C\frac{5\lambda^2+4\lambda+3}{\lambda-z}d\lambda$, where $C=\{z:|z|=3\}$, find $f'(-1+i)$.

5. Given $f(z)=\int\limits_C\frac{3\xi^2+4\xi+5}{\xi-z}d\xi$, where $C=\{z:|z|=4\}$, find $f'(2+i)$.

6. $f(z)=\int\limits_C\frac{2\lambda^2+4\lambda+1}{\lambda-z}d\lambda$

7. $\mathrm{Res}\left(\frac{4z^2}{(z^2+1)^2},i\right)$

8. $f(z)=\frac{\sin^2 z}{(z-1)(z+1)}$ find $\mathrm{Res}(f(z),1)+\mathrm{Res}(f(z),-1)$.

9. Given $f(z)=\frac{1+\sin^2 z}{(z-1)(z+1)}$, find $\mathrm{Res}(f(z),1)+\mathrm{Res}(f(z),-1)$.

$\int_{-\infty}^{+\infty}\frac{2\cos x}{x^2+4x+5}dx$ $=\frac{2\pi}{e}\cos 2$

III. Verifications

1. Show that if $f^{(k)}(z)\equiv 0(\forall z\in C)$, then $f(z)$ is a polynomial of order $<k$.

2. Show that $\lim\limits_{R\to+\infty}\int\limits_{C_R}\frac{7z^2+9}{z^4+7z^2+12}dz=0$, where C_R is the circle centered at 0 with radius R.

$$\lim_{R\to+\infty}\int_{C_R}\frac{z^2+1}{z^4+5z^2+6}dz=0,\ \lim_{R\to+\infty}\int_{C_R}\frac{z^2+3}{z^4+3z^2+2}dz=0$$

3. Show that the equation $z^4-5z^2+2z-1=0$ has just two roots in the

unite disk

1. Show that the function $f(z)=(z^2-2)e^{-x}e^{-iy}$

is an entire function.

2.Suppose that f is analytic and $|f|$ is a constant on a domain a domain D, prove that $f(z)=a$ for some constant a and all $z\in D$.

2. Show that if $f^{(m)}(z)\equiv 0(\forall z\in C)$, then $f(z)$ is a polynomial of order $<m$.

4. 3.Show that the equation $z^4-7z^3+2z^2=1-z$ has just three roots in the unite disk. Show that the equation $z^8-4z^3+z-1=0$ has just three roots in the unite disk.