郑州轻工业学院 2013—2014 学年 第二学期 复变函数 试卷

专业年级及班级________ 姓名________ 学号________

装 订 线

考试类别[学生填写]（□正考 □补考 □重修 □补修 □缓考 □其它）

题号	I	II	III	IV	V	VI					总分
得分											
评阅人											

I. Multiple Choice ($4\times 4=16$ Points)

1. $f(z)=x^2+iy^2$, then which of the following is right ? (　　)

(A) f is analytic at $(0,0)$　　(B) f is differentiable on $y=x$

(C) f is analytic on $y=x$　　(D) f is analytic in the complex plane **C**

2. $f(z)=\dfrac{\cos z-1}{z^5}$, then $z=0$ is (　　)

(A) a pole of order 5　　(B) a pole of order 4

(C) a pole of order 3　　(D) a removable singular point

3. If u(x,y) is harmonic in D, then which of the following function is analytic in D (　　)

(A) $f=\dfrac{\partial u}{\partial x}-i\dfrac{\partial u}{\partial y}$　　(B) $f=\dfrac{\partial u}{\partial x}+i\dfrac{\partial u}{\partial y}$

(C) $f=\dfrac{\partial u}{\partial y}-i\dfrac{\partial u}{\partial x}$　　(D) $f=\dfrac{\partial u}{\partial x}$

4. Which one in the following series is absolutely convergent (　　)

(A) $\sum_{i=1}^{\infty}\dfrac{i^n}{n}$　　(B) $\sum_{i=1}^{\infty}\dfrac{i^n}{n^2}$

(C) $\sum_{i=1}^{\infty}(1+i)^n$　　(D) $\sum_{i=1}^{\infty}(\dfrac{1}{2^2}+\dfrac{i}{n})$

II. Cloze（$4\times 8=32$ Points）

1. Re $(\dfrac{-1+3i}{2-i})=$ ____________,　　Im $(\dfrac{-1+3i}{2-i})=$____________.

2. $3^i=$____________________.

3. $f(z)=x^3+3x^2yi-3xy^2-y^3i$, then $f'(z)=$______________.

4. $\text{Res}[\dfrac{1}{z^2\sin z},0]=$ __________________.

5. The radius of convergence of $\sum_{n=1}^{\infty}\dfrac{n!}{n^n}z^n$ is ___________.

6. If Re $f(z)=2x-x^3+3xy^2$, then f(z) =________________________.

7. $\int_{|z|=1}\dfrac{e^z}{z}dz=$ __________________.

8. The fractional linear function that maps the points $z_1=-1, z_2=0, z_3=1$ onto $w_1=-i, w_2=1, w_3=i$ is ____________________.

III. Calculate the following integrals (28 Points)

1、**(6 Points)** $\int_{|z|=2}\dfrac{z^3+z^2}{(z-1)^3}dz$.

2. **(6 Points)** $\int_{|z|=3}\frac{e^z}{(z-1)^2(z+2)}dz$

3、**(8 Points)** $\int_0^{2\pi}\frac{1}{1+a\cos\theta}d\theta,\ (-1<a<1)..$

4、**(8 Points)** $\int_{-\infty}^{+\infty}\frac{2\cos x}{x^2+4x+5}dx$

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Ⅳ. **Verifications (7 Points)** Determine the number of zeros of $z^9 - 2z^6 + z^2 - 8z - 2 = 0$ inside $|z| < 1$.

Ⅴ. **Verifications (7 Points)** Find the conformal mapping that maps $\{z \mid 0 < \operatorname{Im} z < \pi\}$ onto unit circle.

Ⅵ. **Verifications (10 Points)** Find the Laurent expansion of $f(z) = \dfrac{1}{z(1-z)^2}$ on the annulus (i) $0 < |z| < 1$, (ii) $0 < |z-1| < 1$